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* Viscosity of Gases :-

Consider a gas flowing along a horizontal surface. The layer in contact with the wall of the surface is at rest. The velocity of the layer increases with increase in distance from the fixed layer.

Viscosity is defined as the tangential force per unit area required to maintain a unit velocity gradient.

$$F = -\eta A \frac{du}{dx} \quad \text{--- (1)}$$

Here η is the coefficient of viscosity and A is the area of the layer and $\frac{du}{dx}$ is the velocity gradient.

The velocity of a molecule at any layer at a height x above the fixed layer $= x \times \frac{du}{dx}$. If the mean free path

of a molecule is λ , then the distance for the first collision to occur $= (n - \lambda)$.

The average velocity of these molecules $= (n - \lambda) \frac{du}{dx}$.

The molecules are moving at random in all directions. Approximately it can be assumed that about one-third of the

molecules are moving along each axis. As the molecules in a given space will be free to move in any direction, one-sixth can be assumed to be moving in one direction and the other one-sixth in the opposite direction.

Let C be the r.m.s. velocity of the molecules, n the number of molecules per cc and m the mass of each molecule. Then, the number of molecules crossing unit area in one direction in one second $= \frac{1}{6} nC$.

$$\text{Mass of the molecule} = \frac{1}{6} mnC = M$$

$$\begin{aligned} \text{Momentum in the forward direction} \\ = M \left[(n-\lambda) \frac{du}{dx} \right] \end{aligned}$$

$$\begin{aligned} \text{Momentum in the opposite direction} \\ = M \left[\{n - (-\lambda)\} \frac{du}{dx} \right] \end{aligned}$$

Total change in momentum in one second

$$F = M(n-\lambda) \frac{du}{dx} - M(n+\lambda) \frac{du}{dx}$$

$$F = -2M\lambda \frac{du}{dx}$$

$$F = -2 \left[\frac{1}{6} mnC \right] \lambda \frac{du}{dx}$$

$$\eta = \frac{1}{3} m n C \lambda \frac{du}{dx} \quad \text{--- (ii)}$$

Comparing (i) and (ii)

$$\eta A \frac{du}{dx} = \frac{1}{3} m n C \lambda \frac{du}{dx}$$

But $A = 1 \text{ sq cm}$

$\therefore \eta = \frac{1}{3} m n C \lambda \quad \text{--- (iii)}$

But $\lambda = \frac{1}{\sqrt{2} n a^2}$

$$\eta = \frac{1}{3} m n C \cdot \frac{1}{\sqrt{2} n a^2}$$

$$\eta = \frac{m C}{3 \sqrt{2} n a^2} \quad \text{--- (iv)}$$

But $C \propto \sqrt{T}$

$\therefore \eta \propto \sqrt{T} \quad \text{--- (v)}$

The coefficient of the viscosity of a gas is directly proportional to the square root of its temperature in K.